rate of technological progress is related the level of human capital. Chapter 15 looks at the sources and effects of technological progress.

V. Pedagogy

Students may be confused by the notion that an increase in the saving rate will increase steady-state consumption per worker, as $IS-LM$ analysis suggests that an increase in the marginal propensity to save will reduce output. The differing results arise in different time frames, and both could be true. It may be worthwhile to clarify the short-run/long-run distinction in the context of this example. Moreover, the dynamic simulation in the text provides some idea of the length of the long run.

VI. Extensions

How could the introduction of human capital create the possibility that a higher saving rate could generate a permanently higher growth rate? The issue turns on whether the production function

$$Y/N = F(K/N,\ H/N)$$

exhibits constant returns to scale in its two arguments, so that if both K/N and H/N are doubled, Y/N also doubles. If so, then accumulation of physical and human capital together can generate ongoing growth. In this case, if saving can accumulate human as well as physical capital, then an increase in the saving rate leads to a permanently higher growth rate.

VII. Observations

Depreciation of the capital stock is necessary for the existence of a steady-state equilibrium in the growth model presented in this chapter. Without depreciation, the model generates a positive but steadily decreasing rate of growth. Note that an increase in the rate of depreciation reduces the steady-state capital stock per worker.

Questions and Problems

1.

a. Uncertain. Assuming public saving equal to zero, $I = S$ and investment rate is equal to the saving rate. This is not true for instance in an open economy.

b. False. A higher saving rate can increase the output per worker, but also at a higher saving rate there will be an equilibrium point where the depreciation of capital is equal to investment.

c. False. Due to the decreasing returns of scale of capital and labour, also without depreciation it is not possible to obtain growth of output forever.

d. Uncertain. According to the golden rule, in the long run a higher saving rate determines a higher output and a lower consumption if $s < 50\%$. If $s > 50\%$ it determines a higher consumption ad a lower output.

e. False. It would increase the saving rate, not the consumption. Moreover, the shift from a pay-as-you-system to a fully funded should be gradual, or the current workers risk to pay twice.

f. True. Tax breaks to the people who save is a possible way that government can use to increase savings. A decrease in taxes directly increases consumption, not the saving rate.

g. Uncertain. Under a certain level of education the government should subsidise education spending, but at higher levels it is not so essential. There is a risk of over qualification without a real increase in productivity and skills.

2. I disagree. A good level of saving is necessary to maintain healthy the economy, as stated by the golden rule.

3. An increase in saving after a decade will increase the growth rate of the economy. After 5 decades, the increase in the growth rate will reach the maximum and the new steady state (See Figure 14.8).

4.

a. In this case the saving rate increases, so output per worker will increase if s remains lower than 50% of output.

b. Other things being equal, output per worker increases because more workers enter in the labour market.

5. Output per worker will increase in the long run because of the increase of public saving. The effect of the shift depends on how the government decides to pay the pensions of the people already retired with the old system (pay-as-you-go). If the government does not want to increase public debt, it will increase taxes. An increase in T will increase private saving but there is a stronger decrease in consumption.

6.

a. In equilibrium: $\frac{K_{t+1}}{N} - \frac{K_t}{N} = 0 = S\frac{Y_t}{N} - \delta\frac{K_t}{N}$

Solving we obtain: $\frac{Y_t}{N} = \frac{\delta}{s}\frac{K_t}{N}$.

Substituting the production function: $0.5\sqrt{\frac{K_t}{N}} = \frac{\delta}{s}\frac{K_t}{N}$.

The equilibrium values will be: $\frac{K^*}{N} = 0.5\left(\frac{s}{\delta}\right)^2$ and $\frac{Y^*}{N} = 0.5\frac{s}{\delta}$.

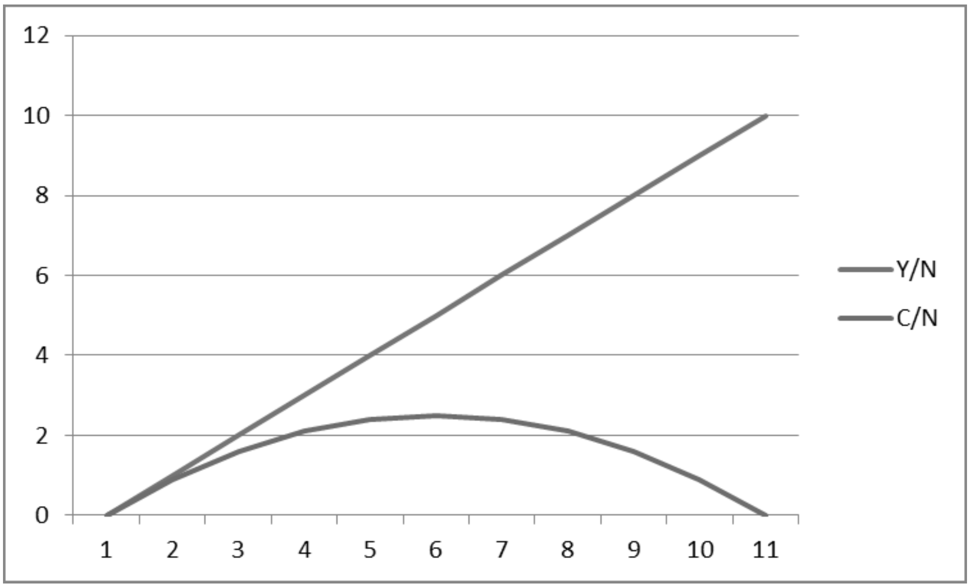
b. From $\frac{Y^*}{N} = 0.5\frac{s}{\delta}$ we can find optimal consumption:

$$\frac{C^*}{N} = \frac{Y^*}{N} - \delta\frac{K^*}{N} = \frac{s(1-s)}{2\delta}$$

c. A higher saving rate determines higher accumulation of capital and higher growth. If saving rate is too high, however, after a peak in $s = 0.5$ consumption declines to zero.

s_t	**y_t**	**c_t**
0	0	0
0.1	1	0.9
0.2	2	1.6
0.3	3	2.1
0.4	4	2.4
0.5	5	2.5
0.6	6	2.4
0.7	7	2.1
0.8	8	1.6
0.9	9	0.9
1	10	0

d.



e. The graph clearly shows that $s = 1$ maximises output per worker, while $s = 0.5$ maximises consumption per worker.

7.

a. Yes, the production function is characterised by constant returns to scale:

$$f(\alpha K, \alpha N) = \alpha f(K, N)$$

$$(\alpha K)^{\frac{1}{3}} (\alpha N)^{\frac{2}{3}} = \alpha K^{\frac{1}{3}} N^{\frac{2}{3}}$$

b. and c. Yes, there are declining returns to K and N.

$$f(\alpha K, N) < \alpha f(K, N)$$

$$(\alpha K)^{\frac{1}{3}} N^{\frac{2}{3}} < \alpha K^{\frac{1}{3}} N^{\frac{2}{3}}$$

$$f(K, \alpha N) < \alpha f(K, N)$$

$$K^{\frac{1}{3}} (\alpha N)^{\frac{2}{3}} < \alpha K^{\frac{1}{3}} N^{\frac{2}{3}}$$

d. Divide each side of the production function per *N*:

$$\frac{Y}{N} = \frac{K^{\frac{1}{3}} N^{\frac{2}{3}}}{N} = \left(\frac{K}{N}\right)^{\frac{1}{3}}$$

e. Substitute the values for *K*/*N* in the steady state:

$$s\frac{Y}{N} = \delta \frac{K}{N}$$

$$s\left(\frac{K}{N}\right)^{\frac{1}{3}} = \delta \frac{K}{N}$$

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^{\frac{3}{2}}$$

f. The steady state will be:

$$\frac{Y^*}{N} = \left(\frac{K^*}{N}\right)^{\frac{1}{3}} = \left(\left(\frac{s}{\delta}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}} = \sqrt{\frac{s}{\delta}}$$

g. The steady state with $s = 0.32$ and $\delta = 0.08$ will be:

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.32}{0.08}} = 2$$

h. The new steady state per worker will be:

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.16}{0.08}} = \sqrt{2}$$

8.

a. The steady state level of capital per worker is:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^{\frac{3}{2}} = \left(\frac{0.1}{0.1}\right)^{\frac{3}{2}} = 1$$

b. The steady state level of output per worker is:

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.1}{0.1}} = 1$$

c. The new steady state of capital per worker and output per worker is:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^{\frac{3}{2}} = \left(\frac{0.1}{0.2}\right)^{\frac{3}{2}} = \sqrt{\frac{1}{8}}$$

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.1}{0.2}} = \sqrt{\frac{1}{2}}$$

d. In $t+1$, $\Delta\delta = 0.1$.

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\left(\frac{K_t}{N}\right)^{\frac{1}{3}} - \delta\frac{K_t}{N}$$

$$\frac{K_{t+1}}{N} = 0.1 - 0.2 + 1 - 0.9$$

$$\frac{K_{t+2}}{N} = 0.1 \times 0.9^{\frac{1}{3}} - 0.2 \times 0.9 + 0.9 = 0.8165$$

$$\frac{K_{t+3}}{N} = 0.01 \times 0.8165^{\frac{1}{3}} - 0.2 \times 0.8165 + 0.8165 = 0.7466$$

The output per worker will be:

$$\frac{Y_{t+1}}{N} = \left(\frac{K_{t+1}}{N}\right)^{\frac{1}{3}} = 0.9^{\frac{1}{3}} = 0.9654$$

$$\frac{Y_{t+2}}{N} = \left(\frac{K_{t+2}}{N}\right)^{\frac{1}{3}} = 0.8165^{\frac{1}{3}} = 0.9346$$

$$\frac{Y_{t+3}}{N} = \left(\frac{K_{t+3}}{N}\right)^{\frac{1}{3}} = 0.7466^{\frac{1}{3}} = 0.9071$$

9.

a. The production function is modified by dividing by *N*:

$$\frac{Y}{N} = \frac{\sqrt{K}\sqrt{N}}{N} = \sqrt{\frac{K}{N}}$$

Now consider the dynamic of capital:

$$K_{t+1} = I_t + (1-\delta)K_t$$

I is the investment at time *t*, δ is the depreciation rate and $(1-\delta)K_t$ is the residual capital from period *t*. Investment is equal to saving and can be written as $I_t = S_t = sY_t$

Also K_{t+1} can be written as:

$$K_{t+1} = sY_t + (1-\delta)K_t$$

Divide by *N* and rearrange:

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N}$$

In equilibrium: $\frac{K_{t+1}}{N} = \frac{K_t}{N}$ so also $s\frac{Y_t}{N} = \delta\frac{K_t}{N}$.

Substitute the production per worker from the production function and solve for the equilibrium value of the capital per worker:

$$s\frac{Y_t}{N} = \delta\frac{K_t}{N}$$

$$s\sqrt{\frac{K}{N}} = \delta\frac{K_t}{N}$$

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2$$

b. $s = 0.15, \delta = 0.075$

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2$$

So, substituting the values:

$$\frac{K^*}{N} = \left(\frac{0.15}{0.075}\right)^2 = 4$$

And the equilibrium value for the output per worker:

$$\frac{Y^*}{N} = \sqrt{\frac{K^*}{N}}$$

$$\frac{Y^*}{N} = \sqrt{4} = 2$$

c. If *s* increases to 0.2:

$$\frac{K^*}{N} = \left(\frac{0.2}{0.075}\right)^2 = \frac{64}{9}$$

$$\frac{Y^*}{N} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

An increase in total saving increases capital accumulation and hence also increases output per worker.