**6.** (a) $x = 50 - \frac{1}{2}y$ (b) $x = \sqrt[5]{y/2}$ (c) $x = \frac{1}{3}[2 + \ln(y/5)]$, defined for $y > 0$

**7.** (a) $y = \ln(2 + e^{x-3})$, defined for $x \in \mathbb{R}$ (b) $y = -\frac{1}{\lambda}\ln a - \frac{1}{\lambda}\ln\left(\frac{1}{x} - 1\right)$, defined for $x \in (0, 1)$

**8.** (a) $\sqrt{13}$ (b) $\sqrt{17}$ (c) $\sqrt{(2-3a)^2} = |2 - 3a|$. (Note that $2 - 3a$ is the correct answer only if $2 - 3a \geq 0$, i.e. $a \leq 2/3$. Check this by putting $a = 3$.)

**9.** $(x-2)^2 + (y+3)^2 = 25$ (b) $(x+2)^2 + (y-2)^2 = 65$

**10.** $(x-3)^2 + (y-2)^2 = (x-5)^2 + (y+4)^2$, which reduces to $x - 3y = 7$. See Fig. A5.R.10.

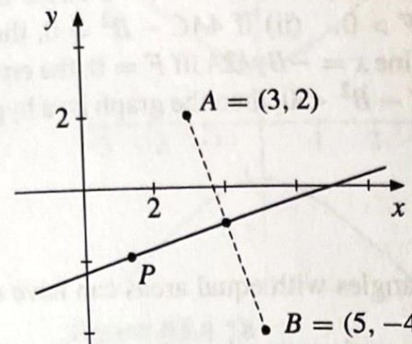


**Figure A5.R.10**

**11.** The function cannot be one-to-one, because at least two persons out of any five must have the same blood group.

# Chapter 6

## 6.1

**1.** $f(3) = 2$. The tangent passes through $(0, 3)$, so has slope $-1/3$. Thus, $f'(3) = -1/3$.

**2.** $g(5) = 1$, $g'(5) = 1$.

## 6.2

**1.** $f(5+h) - f(5) = 4(5+h)^2 - 4 \cdot 5^2 = 4(25 + 10h + h^2) - 100 = 40h + 4h^2$. So $[f(5+h) - f(5)]/h = 40 + 4h \to 40$ as $h \to 0$. Hence, $f'(5) = 40$. This accords with (6) when $a = 4$ and $b = c = 0$.

**2.** (a) $f'(x) = 6x + 2$ (b) $f'(0) = 2$, $f'(-2) = -10$, $f'(3) = 20$. The tangent equation is $y = 2x - 1$.

**3.** $dD(P)/dP = -b$

**4.** $C'(x) = 2qx$

**5.** $$\frac{f(x+h) - f(x)}{h} = \frac{1/(x+h) - 1/x}{h} = \frac{x - (x+h)}{hx(x+h)} = \frac{-h}{hx(x+h)} = \frac{1}{x(x+h)} \xrightarrow[h\to 0]{} \frac{1}{x^2}$$

**6.** (a) $f'(0) = 3$ (b) $f'(1) = 2$ (c) $f'(3) = -1/3$ (d) $f'(0) = -2$ (e) $f'(-1) = 0$ (f) $f'(1) = 4$

**7.** (a) $f(x+h) - f(x) = a(x+h)^2 + b(x+h) + c - (ax^2 + bx + c) = 2ahx + bh + ah^2$, so $[f(x+h) - f(x)]/h = 2ax + b + ah \to 2ax + b$ as $h \to 0$. Thus $f'(x) = 2ax + b$.
(b) $f'(x) = 0$ for $x = -b/2a$. The tangent is parallel to the $x$-axis at the minimum/maximum point.

8. $f'(a) < 0,\ f'(b) = 0,\ f'(c) > 0,\ f'(d) < 0$

9. (a) Expand the left-hand side. (b) Rearrange the identity in (a).
(c) Letting $h \to 0$, the formula follows. (Recall that $\sqrt{x} = x^{1/2}$ and $1/\sqrt{x} = x^{-1/2}$.)

10. (a) $f'(x) = 3ax^2 + 2bx + c$. (b) Put $a = 1$ and $b = c = d = 0$ to get the result in Example 6.2.2. Then put $a = 0$ to get a quadratic expression as in Exercise 7(a).

11. $\dfrac{(x+h)^{1/3} - x^{1/3}}{h} = \dfrac{1}{(x+h)^{2/3} + (x+h)^{1/3}x^{1/3} + x^{2/3}} \to \dfrac{1}{3x^{2/3}}$ as $h \to 0$, and $\dfrac{1}{3x^{2/3}} = \dfrac{1}{3}x^{-2/3}$.

## 6.3

1. $f'(x) = 2x - 4$, so $f(x)$ is decreasing in $(-\infty, 2]$, increasing in $[2, \infty)$.

2. $f'(x) = -3x^2 + 8x - 1 = -3(x - x_0)(x - x_1)$, where $x_0 = \frac{1}{3}(4 - \sqrt{13}) \approx 0.13$ and $x_1 = \frac{1}{3}(4 + \sqrt{13}) \approx 2.54$. Then $f(x)$ is decreasing in $(-\infty, x_0]$, increasing in $[x_0, x_1]$, and decreasing in $[x_1, \infty)$.

3. The expression in the bracket is a sum of two squares, so it is never negative and it is 0 only if both $x_1 + \frac{1}{2}x_2$ and $x_2$ are equal to 0. This happens only when $x_1 = x_2 = 0$. Thus the bracket is always positive if $x_1 \neq x_2$, and then $x_2^3 - x_1^3$ will have the same sign as $x_2 - x_1$. It follows that $f$ is strictly increasing.

## 6.4

1. $C'(100) = 203$ and $C'(x) = 2x + 3$.

2. Here $c$ is the marginal cost, and also the (constant) incremental cost of producing each additional unit, whereas $\bar{C}$ is the fixed cost.

3. (a) $S'(Y) = s$ (b) $S'(Y) = 0.1 + 0.0004Y$

4. $T'(y) = t$, so the marginal tax rate is constant.

5. The interpretation of $x'(0) = -3$ is that at time $t = 0$, the rate of extraction is 3 barrels per minute.

6. (a) $C'(x) = 3x^2 - 180x + 7500$ (b) By (4.6.3), the quadratic function $C'(x)$ has a minimum at $x = 180/6 = 30$.

7. (a) $\pi'(Q) = 24 - 2Q$, and $Q^* = 12$. (b) $R'(Q) = 500 - Q^2$ (c) $C'(Q) = -3Q^2 + 428.4Q - 7900$

8. (a) $C'(x) = 2a_1x + b_1$ (b) $C'(x) = 3a_1x^2$

## 6.5

1. (a) 3 (b) $-1/2$ (c) $13^3 = 2197$ (d) 40 (e) 1 (f) $-3/4$

2. (a) 0.6931 (b) 1.0986 (c) 0.4055 (Actually, using the result in Example 7.12.2, the precise values of these three limits are $\ln 2$, $\ln 3$, and $\ln(3/2)$, respectively.)

3. (a) We have the following table (where $*$ denotes undefined):

| $x$ | 0.9 | 0.99 | 0.999 | 1 | 1.001 | 1.01 | 1.1 |
|---|---|---|---|---|---|---|---|
| $\dfrac{x^2+7x-8}{x-1}$ | 8.9 | 8.99 | 8.999 | $*$ | 9.001 | 9.01 | 9.1 |

(b) $x^2 + 7x - 8 = (x-1)(x+8)$, so $(x^2 + 7x - 8)/(x - 1) = x + 8 \to 9$ as $x \to 1$.

4. (a) 5 (b) 1/5 (c) 1 (d) $-2$ (e) $3x^2$ (f) $h^2$

**5.** (a) $1/6$ (b) $-\infty$ (the limit does not exist). (c) 2 (d) $\sqrt{3}/6$ (e) $-2/3$ (f) $1/4$

**6.** (a) 4 (b) 5 (c) 6 (d) $2a+2$ (e) $2a+2$ (f) $4a+4$

**7.** (a) $x^3-8=(x-2)(x^2+2x+4)$, so the limit is $1/6$. (b) $\lim_{h\to 0}[\sqrt[3]{27+h}-3]/h = \lim_{u\to 3}(u-3)/(u^3-27)$, and $u^3-27=(u-3)(u^2+3u+9)$, so the limit is $\lim_{u\to 3} 1/(u^2+3u+9)=1/27$.
(c) $x^n-1=(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)$, so the limit is $n$.

## 6.6

**1.** (a) 0 (b) $4x^3$ (c) $90x^9$ (d) 0 (Remember that $\pi$ is a constant!)

**2.** (a) $2g'(x)$ (b) $-\frac{1}{6}g'(x)$ (c) $\frac{1}{3}g'(x)$

**3.** (a) $6x^5$ (b) $33x^{10}$ (c) $50x^{49}$ (d) $28x^{-8}$ (e) $x^{11}$ (f) $4x^{-3}$ (g) $-x^{-4/3}$ (h) $3x^{-5/2}$

**4.** (a) $8\pi r$ (b) $A(b+1)y^b$ (c) $(-5/2)A^{-7/2}$

**5.** In (6.2.1) (the definition of the derivative), choose $h=x-a$ so that $a+h$ is replaced by $x$, and $h\to 0$ implies $x\to a$. For $f(x)=x^2$ we get $f'(a)=2a$.

**6.** (a) $F(x)=\frac{1}{3}x^3+C$ (b) $F(x)=x^2+3x+C$ (c) $F(x)=x^{a+1}/(a+1)+C$. (In all cases $C$ is an arbitrary constant.)

**7.** (a) With $f(x)=x^2$ and $a=5$, one has $\lim\limits_{h\to 0}\dfrac{(5+h)^2-5^2}{h}=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}=f'(a)=f'(5)$.

On the other hand, $f'(x)=2x$, so $f'(5)=10$, and the limit is 10.
(b) Let $f(x)=x^5$. Then $f'(x)=5x^4$, and the limit is equal to $f'(1)=5\cdot 1^4=5$.
(c) Let $f(x)=5x^2+10$. Then $f'(x)=10x$, and this is the value of the limit.

## 6.7

**1.** (a) 1 (b) $1+2x$ (c) $15x^4+8x^3$ (d) $32x^3+x^{-1/2}$ (e) $\frac{1}{2}-3x+15x^2$ (f) $-21x^6$

**2.** (a) $\frac{6}{5}x-14x^6-\frac{1}{2}x^{-1/2}$ (b) $4x(3x^4-x^2-1)$ (c) $10x^9+5x^4+4x^3-x^{-2}$. (In (b) and (c), first expand and then differentiate.)

**3.** (a) $-6x^{-7}$ (b) $\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-3/2}$ (c) $-\frac{3}{2}x^{-5/2}$ (d) $-2/(x-1)^2$ (e) $-4x^{-5}-5x^{-6}$ (f) $34/(2x+8)^2$ (g) $-33x^{-12}$ (h) $(-3x^2+2x+4)/(x^2+x+1)^2$

**4.** (a) $\dfrac{3}{2\sqrt{x}(\sqrt{x}+1)^2}$ (b) $\dfrac{4x}{(x^2+1)^2}$ (c) $\dfrac{-2x^2+2}{(x^2-x+1)^2}$

**5.** (a) $f'(L^*)<f(L^*)/L^*$. See Fig. A6.7.5. The tangent at $P$ has the slope $f'(L^*)$. We "see" that the tangent at $P$ is less steep than the straight line from the origin to $P$, which has the slope $f(L^*)/L^*=g(L^*)$. (The inequality follows directly from the characterization of differentiable concave functions in Eq. (8.4.3).)
(b) $\dfrac{d}{dL}\left(\dfrac{f(L)}{L}\right)=\dfrac{1}{L}\left[f'(L)-\dfrac{f(L)}{L}\right]$, as in Example 6.7.7.

**6.** (a) $[2,\infty)$ (b) $[-\sqrt{3},0]$ and $[\sqrt{3},\infty)$ (c) $[-\sqrt{2},\sqrt{2}]$ (d) $(-\infty,\frac{1}{2}(-1-\sqrt{5})]$ and $[0,\frac{1}{2}(-1+\sqrt{5})]$.

**7.** (a) $y=-3x+4$ (b) $y=x-1$ (c) $y=(17x-19)/4$ (d) $y=-(x-3)/9$

**8.** $\dot{R}(t)=\dot{p}(t)x(t)+p(t)\dot{x}(t)$. Here, $R(t)$ increases for two reasons. First, $R(t)$ increases because of the price increase. This increase is proportional to the amount of extraction $x(t)$ and is equal to $\dot{p}(t)x(t)$. But $R(t)$ also rises because

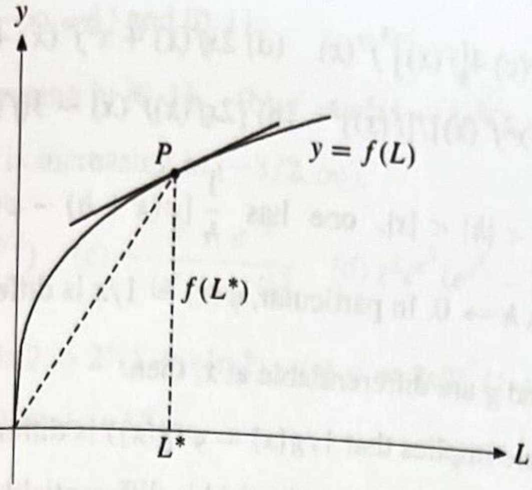


Figure A6.7.5

extraction increases. Its contribution to the rate of change of $R(t)$ must be proportional to the price, and is equal to $p(t)\dot{x}(t)$. In the end $\dot{R}(t)$, the total rate of change of $R(t)$, is the sum of these two parts.

9. (a) $(ad-bc)/(ct+d)^2$ (b) $a(n+\frac{1}{2})t^{n-1/2}+nbt^{n-1}$ (c) $-(2at+b)/(at^2+bt+c)^2$

10. The product rule yields $f'(x)\cdot f(x)+f(x)\cdot f'(x)=1$, so $2f'(x)\cdot f(x)=1$. Hence, $f'(x)=1/2f(x)=1/2\sqrt{x}$.

11. If $f(x)=1/x^n$, the quotient rule yields $f'(x)=(0\cdot x^n-1\cdot nx^{n-1})/(x^n)^2=-nx^{-n-1}$, which is the power rule.

## 6.8

1. (a) $dy/dx=(dy/du)(du/dx)=20u^{4-1}\,du/dx=20(1+x^2)^3 2x=40x(1+x^2)^3$
(b) $dy/dx=(1-6u^5)\,(du/dx)=(-1/x^2)(1-6(1+1/x)^5)$

2. (a) $dY/dt=(dY/dV)(dV/dt)=(-3)5(V+1)^4t^2=-15t^2(t^3/3+1)^4$
(b) $dK/dt=(dK/dL)(dL/dt)=AaL^{a-1}b=Aab(bt+c)^{a-1}$

3. (a) $y'=-5(x^2+x+1)^{-6}(2x+1)$ (b) $y'=\frac{1}{2}\left[x+(x+x^{1/2})^{1/2}\right]^{-1/2}\left(1+\frac{1}{2}(x+x^{1/2})^{-1/2}(1+\frac{1}{2}x^{-1/2})\right)$
(c) $y'=ax^{a-1}(px+q)^b+x^abp(px+q)^{b-1}=x^{a-1}(px+q)^{b-1}[(a+b)px+aq]$

4. $(dY/dt)_{t=t_0}=(dY/dK)_{t=t_0}\cdot(dK/dt)_{t=t_0}=Y'(K(t_0))K'(t_0)$

5. $dY/dt=F'(h(t))\cdot h'(t)$

6. $x=b-\sqrt{ap-c}=b-\sqrt{u}$, with $u=ap-c$. Then $\dfrac{dx}{dp}=-\dfrac{1}{2\sqrt{u}}u'=-\dfrac{a}{2\sqrt{ap-c}}$

7. (i) $h'(x)=f'(x^2)2x$ (ii) $h'(x)=f'(x^ng(x))(nx^{n-1}g(x)+x^ng'(x))$

8. $b(t)$ is the total fuel consumption after $t$ hours. Then $b'(t)=B'(s(t))s'(t)$, so the rate of fuel consumption per hour is equal to the rate per kilometre multiplied by the speed in kph.

9. $dC/dx=q(25-\frac{1}{2}x)^{-1/2}$

10. (a) $y'=5(x^4)^4\cdot 4x^3=20x^{19}$ (b) $y'=3(1-x)^2(-1)=-3+6x-3x^2$

11. (a) (i) $g(5)$ is the amount accumulated if the interest rate is 5% per year, which is approximately €1629.
(ii) $g'(5)$ is the increase in this value per unit increase in the interest rate, which is approximately €155.
(b) $g(p)=1000(1+p/100)^{10}$, so $g(5)=1000\cdot 1.05^{10}=1\,628.89$ to the nearest eurocent.
Moreover, $g'(p)=1000\cdot 10(1+p/100)^9\cdot 1/100$, so $g'(5)=100\cdot 1.05^9=155.13$ to the nearest eurocent.

**12.** (a) $1+f'(x)$ (b) $2f(x)f'(x)-1$ (c) $4[f(x)]^3f'(x)$ (d) $2xf(x)+x^2f'(x)+3[f(x)]^2f'(x)$ (e) $f(x)+xf'(x)$ (f) $f'(x)/[2\sqrt{f(x)}]$ (g) $[2xf(x)-x^2f'(x)]/[f(x)]^2$ (h) $[2xf(x)f'(x)-3(f(x))^2]/x^4$

**13.** (a) Provided that $x \neq 0$ and $0<|h|<|x|$, one has $\frac{1}{h}[\varphi(x+h)-\varphi(x)] = \frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right) = \frac{-h}{h(x+h)x} = \frac{-1}{(x+h)x}$, which tends to $-1/x^2$ as $h \to 0$. In particular, $\varphi(x)=1/x$ is differentiable if $x \neq 0$.

(b) For any $x$ with $g(x) \neq 0$, if $f$ and $g$ are differentiable at $x$, then:

(i) combining (a) with the chain rule implies that $1/g(x) = \varphi(g(x))$ is differentiable at $x$;

(ii) the product rule implies that $f(x)/g(x) = f(x) \cdot [1/g(x)]$ is differentiable at $x$.

## 6.9

**1.** (a) $y''=20x^3-36x^2$ (b) $y''=(-1/4)x^{-3/2}$
(c) $y'=20x(1+x^2)^9$, and then $y''=20(1+x^2)^9+20x\cdot 9\cdot 2x(1+x^2)^8=20(1+x^2)^8(1+19x^2)$

**2.** $\mathrm{d}^2y/\mathrm{d}x^2=(1+x^2)^{-1/2}-x^2(1+x^2)^{-3/2}=(1+x^2)^{-3/2}$

**3.** (a) $y''=18x$ (b) $Y'''=36$ (c) $\mathrm{d}^3z/\mathrm{d}t^3=-2$ (d) $f^{(4)}(1)=84\,000$

**4.** $g'(t)=\frac{2t(t-1)-t^2}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}$, and $g''(t)=\frac{2}{(t-1)^3}$, so $g''(2)=2$.

**5.** With simplified notation: $y'=f'g+fg'$, $y''=f''g+f'g'+f'g'+fg''=f''g+2f'g'+fg''$,
and $y'''=f'''g+f''g'+2f''g'+2f'g''+f'g''+fg'''=f'''g+3f''g'+3f'g''+fg'''$.

**6.** $L=(2t-1)^{-1/2}$, so $\mathrm{d}L/\mathrm{d}t=-\frac{1}{2}\cdot 2(2t-1)^{-3/2}=-(2t-1)^{-3/2}$, and $\mathrm{d}^2L/\mathrm{d}t^2=3(2t-1)^{-5/2}$.

**7.** (a) $R=0$ (b) $R=1/2$ (c) $R=3$ (d) $R=\rho$

**8.** Because $g(u)$ is not concave.

**9.** The Secretary of Defense: $P'<0$. Representative Gray: $P'\geq 0$ and $P''<0$.

**10.** $\mathrm{d}^3L/\mathrm{d}t^3>0$

## 6.10

**1.** (a) $y'=e^x+2x$ (b) $y'=5e^x-9x^2$ (c) $y'=(1\cdot e^x-xe^x)/e^{2x}=(1-x)e^{-x}$
(d) $y'=[(1+2x)(e^x+1)-(x+x^2)e^x]/(e^x+1)^2=[1+2x+e^x(1+x-x^2)]/(e^x+1)^2$
(e) $y'=-1-e^x$ (f) $y'=x^2e^x(3+x)$ (g) $y'=e^x(x-2)/x^3$ (h) $y'=2(x+e^x)(1+e^x)$

**2.** (a) $dx/dt=(b+2ct)e^t+(a+bt+ct^2)e^t=(a+b+(b+2c)t+ct^2)e^t$

(b) $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{3qt^2te^t-(p+qt^3)(1+t)e^t}{t^2e^{2t}}=\frac{-qt^4+2qt^3-pt-p}{t^2e^t}$

(c) $\frac{\mathrm{d}x}{\mathrm{d}t}=[2(at+bt^2)(a+2bt)e^t-(at+bt^2)^2e^t]/(e^t)^2=[t(a+bt)(-bt^2+(4b-a)t+2a)]e^{-t}$

**3.** (a) $y'=-3e^{-3x}$ and $y''=9e^{-3x}$ (b) $y'=6x^2e^{x^3}$ and $y''=6xe^{x^3}(3x^3+2)$
(c) $y'=-x^{-2}e^{1/x}$ and $y''=x^{-4}e^{1/x}(2x+1)$ (d) $y'=5(4x-3)e^{2x^2-3x+1}$ and $y''=5e^{2x^2-3x+1}(16x^2-24x+13)$

4. (a) $(-\infty, \infty)$ (b) $[0, 1/2]$ (c) $(-\infty, -1]$ and $[0, 1]$

5. (a) $y' = 2xe^{-2x}(1-x)$, so $y$ is increasing in $[0, 1]$. (b) $y' = e^x(1 - 3e^{2x})$, so $y$ is increasing in $(-\infty, -\frac{1}{2}\ln 3]$.
(c) $y' = (2x+3)e^{2x}/(x+2)^2$, so $y$ is increasing in $[-3/2, \infty)$.

6. (a) $e^{e^x}e^x = e^{e^x+x}$ (b) $\frac{1}{2}(e^{t/2} - e^{-t/2})$ (c) $\dfrac{e^t - e^{-t}}{(e^t + e^{-t})^2}$ (d) $z^2e^{z^3}(e^{z^3} - 1)^{-2/3}$

7. (a) $y' = 5^x \ln 5$ (b) $y' = 2^x + x2^x \ln 2 = 2^x(1 + x\ln 2)$ (c) $y' = 2x2^{x^2}(1 + x^2 \ln 2)$
(d) $y' = e^x10^x + e^x10^x \ln 10 = e^x10^x(1 + \ln 10)$

## 6.11

1. (a) $y' = 1/x + 3$ and $y'' = -1/x^2$ (b) $y' = 2x - 2/x$ and $y'' = 2 + 2/x^2$
(c) $y' = 3x^2 \ln x + x^2$ and $y'' = x(6\ln x + 5)$ (d) $y' = (1 - \ln x)/x^2$ and $y'' = (2\ln x - 3)/x^3$

2. (a) $x^2 \ln x(3\ln x + 2)$ (b) $x(2\ln x - 1)/(\ln x)^2$ (c) $10(\ln x)^9/x$ (d) $2\ln x/x + 6\ln x + 18x + 6$

3. (a) $1/(x\ln x)$ (b) $-x/(1-x^2)$ (c) $e^x(\ln x + 1/x)$ (d) $e^{x^3}(3x^2 \ln x^2 + 2/x)$ (e) $e^x/(e^x + 1)$
(f) $(2x+3)/(x^2+3x-1)$ (g) $-2e^x(e^x - 1)^{-2}$ (h) $(4x-1)e^{2x^2-x}$

4. (a) $x > -1$ (b) $1/3 < x < 1$ (c) $x \neq 0$

5. (a) $|x| > 1$ (b) $x > 1$ (c) $x \neq e^e$ and $x > 1$

6. (a) $y$ is defined only in $(-2, 2)$, where $y' = -8x/(4 - x^2) > 0$ iff $x < 0$. Thus, $y$ is increasing in $(-2, 0]$.
(b) $y$ is defined for $x > 0$, where $y' = x^2(3\ln x + 1) > 0$ iff $\ln x > -1/3$. Thus, $y$ is increasing in $[e^{-1/3}, \infty)$.
(c) $y$ is defined for $x > 0$, where $y' = (1 - \ln x)(\ln x - 3)/2x^2 > 0$ iff $1 < \ln x < 3$. Thus, $y$ is increasing in $[e, e^3]$.

7. (a) (i) $y = x - 1$ (ii) $y = 2x - 1 - \ln 2$ (iii) $y = x/e$ (b) (i) $y = x$ (ii) $y = 2ex - e$ (iii) $y = -e^{-2}x - 4e^{-2}$

8. (a) $f'(x)/f(x) = 2\ln x + 2$ (b) $f'(x)/f(x) = 1/(2x-4) + 2x/(x^2+1) + 4x^3/(x^4+6)$
(c) $f'(x)/f(x) = -2/[3(x^2-1)]$

9. (a) $(2x)^x(1 + \ln 2 + \ln x)$ (b) $x^{\sqrt{x}-\frac{1}{2}}\left(\frac{1}{2}\ln x + 1\right)$ (c) $\frac{1}{2}(\sqrt{x})^x(\ln x + 1)$

10. $\ln y = v\ln u$, so $y'/y = v'\ln u + vu'/u$ and therefore $y' = u^v(v'\ln u + vu'/u)$.
(Alternatively, note that $y = (e^{\ln u})^v = e^{v\ln u}$, and then use the chain rule.)

11. (a) Let $f(x) = e^x - (1 + x + \frac{1}{2}x^2)$. Then $f(0) = 0$ and $f'(x) = e^x - (1 + x) > 0$ for all $x > 0$, as shown in the exercise. Hence $f(x) > 0$ for all $x > 0$, and the inequality follows.
(b) Consider the two functions $f_1(x) = \ln(1+x) - \frac{1}{2}x$ and $f_2(x) = x - \ln(1+x)$. For more details, see SM.
(c) Consider the function $g(x) = 2(\sqrt{x} - 1) - \ln x$. For more details, see SM.

## Review exercises for Chapter 6

1. $[f(x+h) - f(x)]/h = [(x+h)^2 - (x+h) + 2 - x^2 + x - 2]/h = [2xh + h^2 - h]/h = 2x + h - 1$.
Therefore $[f(x+h) - f(x)]/h \to 2x - 1$ as $h \to 0$, so $f'(x) = 2x - 1$.

2. $[f(x+h) - f(x)]/h = -6x^2 + 2x - 6xh - 2h^2 + h \to -6x^2 + 2x$ as $h \to 0$, so $f'(x) = -6x^2 + 2x$.

3. (a) $y' = 2$, $y'' = 0$ (b) $y' = 3x^8$, $y'' = 24x^7$ (c) $y' = -x^9$, $y'' = -9x^8$ (d) $y' = 21x^6$, $y'' = 126x^5$
(e) $y' = 1/10$, $y'' = 0$ (f) $y' = 5x^4 + 5x^{-6}$, $y'' = 20x^3 - 30x^{-7}$ (g) $y' = x^3 + x^2$, $y'' = 3x^2 + 2x$
(h) $y' = -x^{-2} - 3x^{-4}$, $y'' = 2x^{-3} + 12x^{-5}$

4. Because $C'(1000) \approx C(1001) - C(1000)$, if $C'(1000) = 25$, the additional cost of producing slightly more than 1000 units is approximately 25 per unit. If the price per unit is fixed at 30, the extra profit from increasing output slightly above 1000 units is approximately $30 - 25 = 5$ per unit.

5. (a) $y = -3$ and $y' = -6x = -6$ at $x = 1$, so $y - (-3) = (-6)(x - 1)$, or $y = -6x + 3$.
(b) $y = -14$ and $y' = 1/2\sqrt{x} - 2x = -31/4$ at $x = 4$, so $y = -(31/4)x + 17$.
(c) $y = 0$ and $y' = (-2x^3 - 8x^2 + 6x)/(x + 3)^2 = -1/4$ at $x = 1$, so $y = (-1/4)(x - 1)$.

6. The additional cost of increasing the area by a small amount from 100 m$^2$ is approximately \$250 per m$^2$.

7. (a) $f(x) = x^3 + x$, so $f'(x) = 3x^2 + 1$. (b) $g'(w) = -5w^{-6}$ (c) $h(y) = y(y^2 - 1) = y^3 - y$, so $h'(y) = 3y^2 - 1$.
(d) $G'(t) = (-2t^2 - 2t + 6)/(t^2 + 3)^2$ (e) $\varphi'(\xi) = (4 - 2\xi^2)/(\xi^2 + 2)^2$ (f) $F'(s) = -(s^2 + 2)/(s^2 + s - 2)^2$

8. (a) $2at$ (b) $a^2 - 2t$ (c) $2x\varphi - 1/2\sqrt{\varphi}$

9. (a) $y' = 20uu' = 20(5 - x^2)(-2x) = 40x^3 - 200x$ (b) $y' = \dfrac{1}{2\sqrt{u}} \cdot u' = \dfrac{-1}{2x^2\sqrt{1/x - 1}}$

10. (a) $dZ/dt = (dZ/du)(du/dt) = 3(u^2 - 1)^2 2u3t^2 = 18t^5(t^6 - 1)^2$
(b) $dK/dt = (dK/dL)(dL/dt) = (1/[2\sqrt{L}])(-1/t^2) = -1/[2t^2\sqrt{1 + 1/t}]$

11. (a) $\dot{x}/x = 2\dot{a}/a + \dot{b}/b$ (b) $\dot{x}/x = \alpha\dot{a}/a + \beta\dot{b}/b$ (c) $\dot{x}/x = (\alpha + \beta)(\alpha a^{\alpha-1}\dot{a} + \beta b^{\beta-1}\dot{b})/(a^\alpha + b^\beta)$

12. $dR/dt = (dR/dS)(dS/dK)(dK/dt) = \alpha S^{\alpha-1}\beta\gamma K^{\gamma-1}Apt^{p-1} = A\alpha\beta\gamma pt^{p-1}S^{\alpha-1}K^{\gamma-1}$

13. (a) $h'(L) = apL^{a-1}(L^a + b)^{p-1}$ (b) $C'(Q) = a + 2bQ$ (c) $P'(x) = ax^{1/q-1}(ax^{1/q} + b)^{q-1}$

14. (a) $y' = -7e^x$ (b) $y' = -6xe^{-3x^2}$ (c) $y' = xe^{-x}(2 - x)$ (d) $y' = e^x[\ln(x^2 + 2) + 2x/(x^2 + 2)]$
(e) $y' = 15x^2e^{5x^3}$ (f) $y' = x^3e^{-x}(x - 4)$ (g) $y' = 10(e^x + 2x)(e^x + x^2)^9$ (h) $y' = 1/2\sqrt{x}(\sqrt{x} + 1)$

15. (a) $[1, \infty)$ (b) $[0, \infty)$ (c) $(-\infty, 1]$ and $[2, \infty)$

16. (a) $\dfrac{d\pi}{dQ} = P(Q) + QP'(Q) - c$ (b) $\dfrac{d\pi}{dL} = PF'(L) - w$

# Chapter 7

## 7.1

1. Differentiating w.r.t. $x$ yields $6x + 2y' = 0$, so $y' = -3x$. Solving the given equation for $y$ yields $y = 5/2 - 3x^2/2$, implying that $y' = -3x$.

2. Implicit differentiation yields $(*)$ $2xy + x^2(dy/dx) = 0$, and so $dy/dx = -2y/x$. Differentiating $(*)$ implicitly w.r.t. $x$ gives $2y + 2x(dy/dx) + 2x(dy/dx) + x^2(d^2y/dx^2) = 0$. Inserting the result for $dy/dx$, and simplifying yields $d^2y/dx^2 = 6y/x^2$. These results follows more easily by differentiating $y = x^{-2}$ twice.

3. (a) $y' = (1 + 3y)/(1 - 3x) = -5/(1 - 3x)^2$ and $y'' = 6y'/(1 - 3x) = -30/(1 - 3x)^3$.
(b) $y' = 6x^5/5y^4 = (6/5)x^{1/5}$ and $y'' = 6x^4y^{-4} - (144/25)x^{10}y^{-9} = (6/25)x^{-4/5}$.