7. Put $u = x+2$ and $v = 3-x$. Then the integral becomes

$$\int_0^5 u^{-1/2}\,du - \int_5^0 v^{-1/2}\,dv = 2\lim_{\varepsilon\to 0}\int_\varepsilon^5 u^{-1/2}\,du = 4\lim_{\varepsilon\to 0}\Big|_\varepsilon^5 u^{1/2} = 4\lim_{\varepsilon\to 0}\left(\sqrt{5}-\sqrt{\varepsilon}\right) = 4\sqrt{5}.$$

8. (a) $z = \int_0^\tau (1/\tau)e^{-rs}\,ds = (1-e^{-r\tau})/r\tau$ (b) $z = \int_0^\tau 2(\tau - s)\tau^{-2}e^{-rs}\,ds = (2/r\tau)\left[1-(1/r\tau)(1-e^{-r\tau})\right]$

9. $\int x^{-2}\,dx = -x^{-1} + C$. So evaluating $\int_{-1}^1 x^{-2}\,dx$ as $\Big|_{-1}^1 -x^{-1}$ gives the nonsensical answer -2.

The error arises because x^{-2} diverges to $+\infty$ as $x\to 0$. (In fact, $\int_{-1}^1 x^{-2}\,dx$ diverges to $+\infty$.)

10. Using the answer to Exercise 10.6.6(b), one has $\int_h^1 (\ln x/\sqrt{x})\,dx = \Big|_h^1 (2\sqrt{x}\ln x - 4\sqrt{x}) = -4-(2\sqrt{h}\ln h - 4\sqrt{h})$. As $h\to 0^+$, l'Hôpital's rule implies that $\sqrt{h}\ln h = \ln h/h^{-1/2} =$ "∞/∞" $\to 0$, so the given integral converges to -4.

11. $\int_1^A [k/x - k^2/(1+kx)]\,dx = k\ln[1/(1/A+k)] - k\ln[1/(1+k)] \to k\ln(1/k) - k\ln[1/(1+k)] = \ln(1+1/k)^k$ as $A\to\infty$. So $I_k = \ln(1+1/k)^k$, which tends to $\ln e = 1$ as $k\to\infty$.

12. The suggested substitution $u = (x-\mu)/\sqrt{2}\sigma$ gives $du = dx/\sigma\sqrt{2}$, and so $dx = \sigma\sqrt{2}\,du$. Hence:

(a) $\int_{-\infty}^{+\infty} f(x)\,dx = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{-u^2}\,du = 1$, by (10.7.9).

(b) $\int_{-\infty}^{+\infty} xf(x)\,dx = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}(\mu+\sqrt{2}\sigma u)e^{-u^2}\,du = \mu$, using part (a) and (10.7.5).

(c) $\int_{-\infty}^{+\infty}(x-\mu)^2 f(x)\,dx = \int_{-\infty}^{+\infty} 2\sigma^2u^2\frac{1}{\sigma\sqrt{2\pi}}e^{-u^2}\sigma\sqrt{2}\,du = \sigma^2\frac{2}{\sqrt{\pi}}\int_{-\infty}^{+\infty}u^2e^{-u^2}\,du$. Now integration by parts yields $\int u^2e^{-u^2}\,du = -\frac{1}{2}ue^{-u^2} + \int \frac{1}{2}e^{-u^2}\,du$, so $\int_{-\infty}^{+\infty} u^2e^{-u^2}\,du = \frac{1}{2}\sqrt{\pi}$. Hence the integral equals σ^2.

Review exercises for Chapter 10

1. (a) $-16x + C$ (b) $5^5x + C$ (c) $3y - \frac{1}{2}y^2 + C$ (d) $\frac{1}{2}r^2 - \frac{16}{5}r^{5/4} + C$ (e) $\frac{1}{9}x^9 + C$ (f) $\frac{2}{7}x^{7/2} + C$. ($x^2\sqrt{x} = x^2\cdot x^{1/2} = x^{5/2}$.) (g) $-\frac{1}{4}p^{-4} + C$ (h) $\frac{1}{4}x^4 + \frac{1}{2}x^2 + C$

2. (a) $e^{2x} + C$ (b) $\frac{1}{2}x^2 - \frac{25}{2}e^{2x/5} + C$ (c) $-\frac{1}{3}e^{-3x} + \frac{1}{3}e^{3x} + C$ (d) $2\ln|x+5| + C$

3. (a) $\int_0^{12} 50\,dx = \Big|_0^{12} 50x = 600$ (b) $\int_0^2 (x - \frac{1}{2}x^2)\,dx = \Big|_0^2(\frac{1}{2}x^2 - \frac{1}{6}x^3) = \frac{2}{3}$

(c) $\int_{-3}^3 (u+1)^2\,du = \Big|_{-3}^3 \frac{1}{3}(u+1)^3 = 24$ (d) $\int_1^5 \frac{2}{z}\,dz = \Big|_1^5 2\ln z = 2\ln 5$

(e) $\int_2^{12}\frac{3}{t+4}\,dt = \Big|_2^{12} 3\ln(t+4) = 3(\ln 16 - \ln 6) = 3\ln(8/3)$

(f) $I = \int_0^4 v\sqrt{v^2+9}\,dv = \Big|_0^4 \frac{1}{3}(v^2+9)^{3/2} = 98/3$. (Or let $z = \sqrt{v^2+9}$, when $z^2 = v^2+9$, so $2z\,dz = 2v\,dv$, or $v\,dv = z\,dz$. When $v = 0$, $z = 3$, and when $v = 4$, $z = 5$, so $I = \int_3^5 z^2\,dz = \Big|_3^5 \frac{1}{3}z^3 = 98/3$.)

4. (a) 5/4 (b) 31/20 (c) -5 (d) $e-2$ (e) 52/9 (f) $\frac{1}{3}\ln(6/5)$ (g) $(1/256)(3e^4+1)$ (h) $2e^{-1}$.

5. (a) $10 - 18\ln(14/9)$. (Substitute $z = 9 + \sqrt{x}$.) (b) $886/15$. (Substitute $z = \sqrt{t+2}$.)
(c) $195/4$. (Substitute $z = \sqrt[3]{19x^3 + 8}$.)

6. (a) $F'(x) = 4(\sqrt{x} - 1)$. (Note that $\int_4^x (u^{1/2} + xu^{-1/2})\, du = \Big|_4^x (\frac{2}{3}u^{3/2} + 2xu^{1/2}) = \frac{8}{3}x^{3/2} - \frac{16}{3} - 4x$.)
(b) Using (10.3.8), $F'(x) = \ln x - (\ln\sqrt{x})(1/2\sqrt{x}) = \ln x - \ln x/4\sqrt{x}$.

7. $C(Y) = 0.69Y + 1000$

8. Integrating the marginal cost function gives $C(x) = C_0 + \int_0^x (\alpha e^{\beta u} + \gamma)\, du = C_0 + \Big|_0^x \frac{\alpha}{\beta}e^{\beta u} = \frac{\alpha}{\beta}(e^{\beta x} - 1) + \gamma x + C_0$.

9. Let $\int_{-1}^{3} f(x)\, dx = A$ and $\int_{-1}^{3} g(x)\, dx = B$. Then $A + B = 6$ and $3A + 4B = 9$, from which we find $A = 15$ and $B = -9$. Then $I = A + B = 6$.

10. (a) $P^* = 70$, $Q^* = 600$. CS = 9000, PS = 18 000. See Fig. A10.R.10a.
(b) $P^* = Q^* = 5$, CS $= 50\ln 2 - 25$, PS = 1.25. See Fig. A10.R.10b.

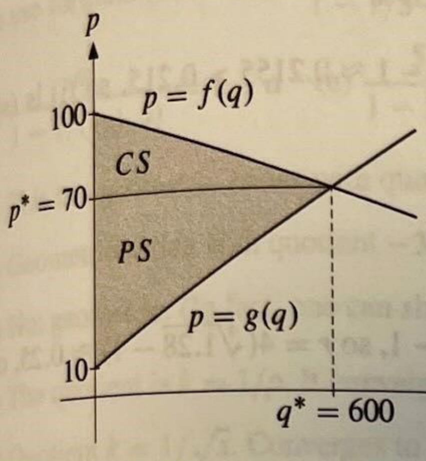

Figure A10.R.10a

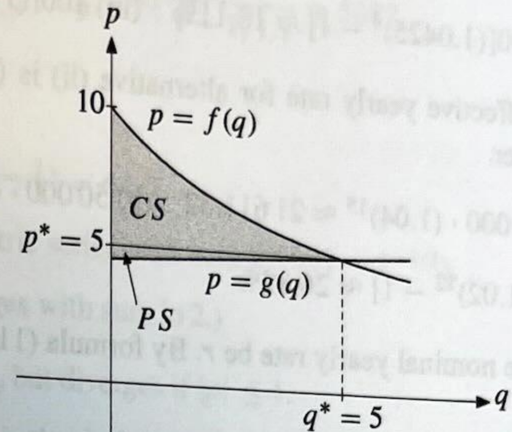

Figure A10.R.10b

11. (a) $f'(t) = 4\ln t(2 - \ln t)/t^2$, $f''(t) = 8[(\ln t)^2 - 3\ln t + 1]/t^3$.
(b) $(e^2, 16/e^2)$ is a local maximum point, $(1, 0)$ is a local (and global) minimum point. See Fig. A10.R.11.
(c) Area = 32/3. (*Hint:* $\int f(t)\, dt = \frac{4}{3}(\ln t)^3 + C$.)

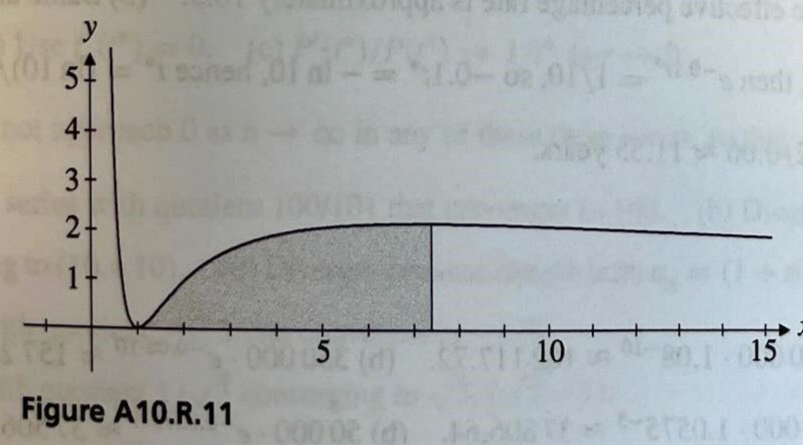

Figure A10.R.11

12. (a) $\int_0^\infty f(r)\, dr = \int_0^\infty (1/m)e^{-r/m}\, dr = 1$, as in Example 10.7.1, and $\int_0^\infty rf(r)\, dr = \int_0^\infty r(1/m)e^{-r/m}\, dr = m$, as in Exercise 10.7.3(a). So mean income is m.
(b) $x(p) = n\int_0^\infty (ar - bp)f(r)\, dr = n\left(a\int_0^\infty rf(r)\, dr - bp\int_0^\infty f(r)\, dr\right) = n(am - bp)$, by the results in part (a).